

# ACTUARIAL VALUATIONS

Remainder, Income,  
and Annuity  
Examples  
— For One Life,  
Two Lives, and  
Terms Certain

For Use in Income,  
Estate, and Gift Tax  
Purposes, Including  
Valuation of Pooled  
Income Fund  
Remainder Interests

## Version 3A

Table S - Based on Life Table 2000CM  
Interest at 0.2 Percent

Age	Annuity	Life Estate	Remainder
55	24.9133	0.04853	0.85816
56	24.0253	0.04805	0.85889
57	23.2464	0.04754	0.86054
58	22.4773	0.04695	0.86221
59	21.7175	0.04633	0.86390
60	20.9662	0.04568	0.86560
61	20.2244	0.04505	0.86731
62	19.4932	0.04439	0.86902
63	18.7731	0.04375	0.87073
64	18.0640	0.04313	0.87246
65	17.3658	0.04252	0.87419
66	16.6737	0.04193	0.87592
67	15.9895	0.04135	0.87766
68	15.3150	0.04078	0.87939
69	14.6513	0.04023	0.88112
70	14.0001	0.03969	0.88284
71	13.3604	0.03916	0.88455
72	12.7332	0.03865	0.88625
73	12.1201	0.03815	0.88795
74	11.5228	0.03766	0.88964
75	10.9430	0.03718	0.89132
76	10.3819	0.03671	0.89301
77	9.8395	0.03625	0.89470
78	9.3157	0.03580	0.89639
79	8.8104	0.03535	0.89808
80	8.3235	0.03491	0.89978
81	7.8550	0.03448	0.90149
82	7.4049	0.03405	0.90320
83	6.9732	0.03363	0.90492
84	6.5598	0.03322	0.90665
85	6.1646	0.03281	0.90837
86	5.7875	0.03241	0.91010
87	5.4284	0.03201	0.91182
88	5.0872	0.03162	0.91355
89	4.7638	0.03123	0.91528
90	4.4581	0.03085	0.91701
91	4.1700	0.03047	0.91874
92	3.8994	0.03010	0.92047
93	3.6462	0.02973	0.92220
94	3.4103	0.02937	0.92393
95	3.1915	0.02901	0.92566
96	2.9897	0.02866	0.92739
97	2.8048	0.02831	0.92912
98	2.6367	0.02796	0.93085
99	2.4844	0.02762	0.93258
100	2.3478	0.02728	0.93431



# ACTUARIAL VALUATIONS

## Version 3A

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For One Life, Two Lives, and Terms Certain.

For Use With Respect to Income, Estate, and  
Gift Tax Purposes,  
Including Valuation of Pooled Income Fund  
Remainder Interests.

Department of the Treasury  
**Internal Revenue Service**  
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# ACTUARIAL TABLES ASSOCIATED WITH PUBLICATION 1457

The actuarial tables associated with this publication are as follows:

<u>Section</u>	<u>Table</u>	<u>Type of Factors</u>
1	Table S	Single Life Factors  2-Life Last to Die Remainder Factors:
2-1	Table R(2)	Interest From: <b>0.2 to 4.0</b> Percent
2-2		<b>4.2 to 8.0</b> Percent
2-3		<b>8.2 to 12.0</b> Percent
2-4		<b>12.2 to 16.0</b> Percent
2-5		<b>16.2 to 20.0</b> Percent
3	Table B	Term Certain Factors
4	Table H	Commutation Factors
5	Table K	Adjustment Factors
6	Table 2000CM	Mortality Table

## USE OF EXAMPLES AND TABLES

This publication sets forth examples for using actuarial factors for certain income, gift, and estate tax valuations of future interests. This publication *does not contain* the tables of actuarial factors used in these examples. The actuarial tables cited in the examples below can be found on the IRS website at the following address:

**Website:** <http://www.irs.gov/retirement/article/0,,id=206601,00.html>

The examples illustrate methods for using the factors in the associated tables for valuations, and for finding other factors not found directly in the tables. The examples in **Part A** show the use of last-to-die remainder factors in Table R(2), and show the method for finding last-to-die life estate and annuity factors. **Part B and Part C** examples illustrate the use of factors from the single life Table S and from the last-to-die Table R(2) to obtain factors for such time as one person survives another, and to obtain first-to-die factors. In **Part D**, commutation factors from Table H are used to find factors for one life and a term of years. **Part E** provides similar explanations to compute factors for two lives and term of years. **Part F** demonstrates the use of the remainder factors for pooled income funds.

The factors and tables associated with this publication involving life contingencies are derived from the values of  $l_x$  taken from the Life Table for the Total Population appearing as Table 1, in "*U.S. Decennial Life Tables for 1999-2001*" published by the U.S. Department of Health and Human Services, Public Health Service, National Center for Health Statistics. Table 1 appears in the associated set of tables in Section 6, labeled as Table 2000CM.

## ASSOCIATED TABLES ON THE WEB

In the IRS actuarial tables on the web associated with this publication and these examples, the factors in Sections 1 through 5 are based on interest rates ranging from 0.2 percent to 20.0 percent in intervals of 0.2 percent. The wide range of rates is shown pursuant to Section 7520 of the Internal Revenue Code, which requires the use of an interest rate of 120 percent of the annual mid-term applicable federal rate for the month in which the valuation date falls. All of the factors associated with this publication reflect annual compounding of interest.

**Table S, Section 1**, contains factors for the present worth of a life annuity, a life estate, and a remainder interest based on a single life.

**Table R(2), Section 2**, contains factors for the present worth of the remainder interest in \$1.00 payable at the death of the last to die of two persons.

**Table B, Section 3**, contains factors for the present worth of an annuity, an income interest, and a remainder interest for a term certain.

**Table H, Section 4**, contains commutation tables for  $D_x$ ,  $N_x^{\circ}$ , and  $M_x^{\text{---}}$ .

**Table K, Section 5**, contains adjustment factors for annuities payable at the end of annual, semi-annual, quarterly, monthly, and weekly periods.

**Table 2000CM, Section 6**, is the underlying mortality table used to calculate factors involving life contingencies.

## Historical Synopsis of Tables

Period	Table	Interest Rate	Publications
1-1-1951 to 12-31-1970	US1938	3.5%	11
1-1-1971 to 11-30-1983	Table LN	6%	723, 723A, 723B
12-1-1983 to 4-30-1989	Table CM	10%	723C, 723D, 723E
5-1-1989 to 4-30-1989*	80CNSMT	§ 7520 rates	1457, 1458, 1459 (5-1989 release)
5-1-1999 to 4-30-2009	90CM	§ 7520 rates	1457, 1458, 1459 (7-1999 release)
5-1-2009 --	2000CM	§ 7520 rates	1457, 1458, 1459 (5-2009 release)

\* On October 22, 1988, section 7520 was enacted, which prescribed the use of an interest rate equal to 120 percent of the midterm applicable federal rate, rounded to the nearest two tenths of a percent for actuarial computations.

## EXAMPLES

### A. Two Life Last-to-Die Factors

**Example 1.** Based on an interest rate of 4.2 percent, the present worth of \$1.00 due at the death of the last to die of two persons aged 60 and 65 is \$0.37309. On page 220 in Table R(2) Section 2-2 will be found the older age 65 and the younger age 60. Across from those ages, under the column headed 4.2 % is the remainder factor 0.37309.

A life estate factor or an annuity factor for the same ages and interest rate can be computed using examples 2 and 3.

**Example 2.** Based on an interest rate of 4.2 percent, the present worth of the right to receive the use of \$1.00 until the death of the last to die of two persons aged 60 and 65 is \$0.62691, determined as follows:

$$\begin{aligned}\text{Remainder Factor from Example 1} &= 0.37309 \\ \text{Income Factor} &= 1.00000 - 0.37309 \\ &= 0.62691\end{aligned}$$

**Example 3.** Based on an interest rate of 4.2 percent, the present worth of an annuity of \$1.00 per annum payable at the end of each year until the death of the last to die of two persons aged 60 and 65 is \$14.9264, determined as follows:

$$\begin{aligned}\text{Income Factor from Example 2} &= 0.62691 \\ \text{Annuity factor} &= 0.62691 \div 0.042 \\ &= 14.9264\end{aligned}$$

### B. Income or Annuity Payable for Such Time as One Person Survives Another

**Example 4.** Based on an interest rate of 4.2 percent, the present worth of the right to receive the use of \$1.00 for such time as a person aged 65 survives a person aged 60 is \$0.07459 determined as follows:

$$\begin{aligned}\text{Income Factor from Example 2} &= 0.62691 \\ \text{Single Life Income Factor from Table S(4.2), age 60} &= 0.55232 \\ \text{Required Income Factor} &= 0.62691 - 0.55232 \\ &= 0.07459\end{aligned}$$



**Example 5.** Based on an interest rate of 4.2 percent, the present worth of an annuity of \$1.00 per annum payable annually at for such time as a person aged 65 survives a person aged 60 is \$1.7760 determined as follows:

$$\begin{aligned}
 \text{Joint and Survivor Annuity Factor from Example 3} &= 14.9264 \\
 \text{Single Life Annuity Factor, age 60, From Table S(4.2)} &= 13.1504 \\
 \text{Required Annuity Factor} &= 14.9264 - 13.1504 \\
 &= 1.7760
 \end{aligned}$$

### C. First-to-Die Factors

**Example 6.** Based on an interest rate of 4.2 percent, the present worth of \$1.00 due at the death of the first to die of two persons aged 60 and 65 is \$0.58839, determined as follows:

$$\begin{aligned}
 \text{Single Life Remainder Factor, aged 60, from Table S(4.2)} &= 0.44768 \\
 \text{Single Life Remainder Factor, aged 65, from Table S(4.2)} &= 0.51377 \\
 \text{Joint and Survivor Remainder Factor from Example 1} &= 0.37309 \\
 \text{Required Remainder Factor} &= 0.44768 + 0.51377 - 0.37309 \\
 &= 0.58836
 \end{aligned}$$

**Example 7.** Based on an interest rate of 4.2 percent, the present worth of the right to receive the use of \$1.00 until the death of the first to die of two persons aged 60 and 65 is \$0.41164, determined as follows:

$$\begin{aligned}
 \text{First-to-die Remainder Factor from Example 6} &= 0.58836 \\
 \text{Income Factor} &= 1.00000 - 0.58836 \\
 &= 0.41164
 \end{aligned}$$

**Example 8.** Based on an interest rate of 4.2 percent, the present worth of an annuity of \$1.00 per annum payable annually until the death of the first to die of two persons aged 60 and 65 is \$9.8010, determined as follows:

$$\begin{aligned}
 \text{Income Factor from Example 7} &= 0.41164 \\
 \text{Required Annuity Factor} &= 0.41164 \div .042 \\
 &= 9.8010
 \end{aligned}$$

## D. Factors Involving One Life and a Term of Years

**Example 9.** Based on an interest rate of 6.8 percent, the present worth of a temporary annuity of \$1.00 per annum payable annually for 10 years or until the prior death of a person aged 60 is \$6.6822, determined as follows:

Initial age	=	60
plus Term of years	=	10
		-----
Terminal age	=	70
N-factor, Table H(6.8), age 60	=	17431.19
minus N-factor, Table H(6.8), age 70	=	6130.086
		-----
Difference	=	11301.104
D-factor, Table H(6.8), age 60	=	1691.236
Required Annuity Factor	=	$11301.104 \div 1691.236$
	=	6.6822

**Example 10.** Based on an interest rate of 6.8 percent, the present worth of \$1.00 due at the death of a person aged 60 provided such death occurs within the first 10 years is \$0.10336, determined as follows:

Initial age	=	60
plus Term of years	=	10
		-----
Terminal age	=	70
M-factor, Table H(6.8), age 60	=	505.9147
minus M-factor, Table H(6.8), age 70	=	331.1154
		-----
Difference	=	174.7993
D-factor, Table H(6.8), age 60	=	1691.236
Required Remainder Factor	=	$174.7993 \div 1691.236$
	=	0.10336

**Example 11.** Based on an interest rate of 6.8 percent, the present worth of the right to receive the income from \$1.00 for 10 years or until the prior death of a person aged 60 is \$0.45439, determined as follows:

Temporary Annuity Factor, Example 9	=	6.6822
Required Income Factor	=	$6.6822 \times .068$
	=	0.45439

**Example 12.** Based on an interest rate of 6.8 percent, the present worth of the right of a person aged 21, if living, to receive \$1.00 upon attaining age 30 is \$0.54953, determined as follows:

$$\begin{aligned} \text{D-factor, Table H(6.8), age 30} &= 13582.48 \\ \text{divided by D-factor, Table H(6.8), age 21} &= 24761.63 \\ \text{Required Factor} &= 0.54853 \end{aligned}$$

**Example 13.** The probability that a person age 21 will be alive at age 30 is 0.991611, determined as follows:

$$\begin{aligned} l_x \text{ value, Table 2000CM, page 693, age 30} &= 97750 \\ l_x \text{ value, Table 2000CM, age 21} &= 98577 \\ \text{Required probability} &= 97750 \div 98577 \\ &= 0.991611 \end{aligned}$$

**Example 14.** Based on an interest rate of 6.8 percent, the present worth of a temporary annuity of \$1.00 per annum payable in equal monthly installments for 10 years or until the prior death of a person aged 60 is \$6.8880, determined as follows:

$$\begin{aligned} \text{Annuity factor from Example 9} &= 6.6822 \\ \text{Monthly Adjustment Factor, Table K, page 691, at 6.8 \%} &= 1.0308 \\ \text{Required annuity factor} &= 6.6822 \times 1.0308 \\ &= 6.8880 \end{aligned}$$

### **E. Factors Involving Two lives and a Term of Years**

**Example 15.** Based on an interest rate of 6.8 percent, the present worth of the right to receive \$1.00 at the end of 10 years provided at least one of two persons aged 60 and 65 alive is \$0.50169, determined as follows:

$I_x$  value, Table 2000CM, age 60 = 87595

$I_x$  value, age 70 = 74794

$$1 - \frac{74794}{87595} = 0.146138$$

$I_x$  value, Table 2000CM, age 65 = 82224

$I_x$  value, age 75 = 64561

$$1 - \frac{64561}{82224} = 0.214816$$

Remainder Factor, Table B(6.8), 10 year term = 0.517950

$$\begin{aligned} \text{Required Remainder Factor} &= (1 - 0.146138 \times 0.214816) \times 0.517950 \\ &= 0.50169 \end{aligned}$$

## F. Pooled Income Funds

Table R(2) may be used to obtain joint and survivor remainder factors for pooled income funds. The interest rate to be used and the find the remainder factor is the yearly rate of return for the fund as defined in I.R.S. Notice 89-60. If the yearly rate of return falls between two interest rates for which the factors are given in Table R(2), a linear interpolation must be made.

**Example 16.** Based on a yearly rate of return of 5.636 percent, the present worth of the remainder interest in a pooled income fund of \$1.00 payable at the death of the last to die of two persons aged 60 and 65 is \$0.27674 determined as follows:

$$\begin{array}{r} \text{Remainder Factor, Table R(2), at 5.6 \%} = 0.27710 \\ \text{minus Remainder Factor, Table R(2), at 5.8 \%} = 0.26590 \\ \hline \text{Difference} = 0.01120 \\ \\ \begin{array}{r} 5.636 \% - 5.600 \% \quad \quad \quad \text{X} \\ \hline 5.800 \% - 5.600 \% \quad \quad \quad 0.01120 \end{array} \\ \\ \text{X} = 0.00202 \\ \\ \begin{array}{r} \text{Remainder factor at 5.6 \%} = 0.27710 \\ \text{minus X} = 0.00202 \\ \hline \text{Required Interpolated Remainder Factor} = 0.27508 \end{array} \end{array}$$